Complete prepotential for five dimensional $\mathcal{N}$=1 superconformal field theory

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Based on the work in progress with Hirotaka Hayashi, Sung-Soo Kim, and Kimyeong Lee

§1. Introduction

5d $\mathcal{N}$=1 gauge theory at Coulomb phase

"IMS Prepotential" [Intriligator, Morrison, Seiberg '97]

- **Perturbatively 1-loop exact**
- **Locally cubic due to gauge invariance**
- **Coefficients of the polynomial changes at the point where the massless particle appears**

IMS Prepotential [Intriligator, Morrison, Seiberg '97]

$$F = \frac{1}{2} m_0 h_{ij} \phi_i \phi_j + \frac{k}{6} d_{ijk} \phi_i \phi_j \phi_k + \frac{1}{12} \left(\underbrace{\sum_{\mathbf{R}} |\mathbf{R} \cdot \phi|^3}_{\text{vector multiplet}} - \underbrace{\sum_f \sum_{\mathbf{w} \in \mathbf{W}_f} |\mathbf{w} \cdot \phi + m_f|^3}_{\text{hypermultiplet}} \right)$$

ϕ_i : Vector multiplet of $U(1)_i$, $\quad m_0 = \dfrac{1}{{g_{YM}}^2}$: Bare coupling

$h_{ab} = \mathrm{Tr}(T_a T_b)$, $\quad d_{abc} = \dfrac{1}{2} \mathrm{Tr}\left(T_a (T_b T_c + T_c T_b)\right)$,

k : Chern-Simons level

R : Root of the gauge group, $\quad$ W_f : Weight of the gauge group

f : Label for the hypermultiplet, $\quad$ m_f : Mass of the hypermultiplet

Question

How about "non-perturbative effect" ?

What if the instanton particle becomes massless?

Conventional Answer (?)

No instanton effect

if

$$m_0 \gg |\langle \phi_i \rangle|, |m_f|$$

Conventional Discussion (?)

5d theory on S¹ (circumference β)

Instanton factor: $e^{-\beta m_0} \to 0 \quad (\text{ as } \beta \to \infty)$

No instanton effect in 5d gauge theory

if $m_0 > 0$

Conventional Discussion (?)

5d theory on S^1 (circumference β)

Instanton effect is suppressed by:

$$e^{-\beta\left(m_0+(\text{ linear combination of } \langle\phi_i\rangle, m_f)\right)} \to 0 \quad (\text{ as } \beta \to \infty)$$

No instanton effect in 5d gauge theory

if $m_0 \gg |\langle\phi_i\rangle|, |m_f|$

Question

How about "non-perturbative effect" ?

What if the instanton particle becomes massless?

Answer in this talk

There is "non-perturbative effect" if we write down the prepotential for whole parameter region,

→ "Complete prepotential"

Does it make sense to consider the region

$$m_0 = \frac{1}{{g_{YM}}^2} < 0 \ ?$$

YES in the following sense

RG flow

UV		**IR**
5d SCFT	**Relevant deformation** e.g. $m_0 > 0$ →	**5d SUSY gauge theory**
	Relevant deformation e.g. $m_0 < 0$ →	**Another gauge theory or a Non-Lagrangian theory**

Nicely described by (*p*,*q*) 5-brane web [Aharony, Hanany, Kol '97]

How to compute the prepotential for

$$m_0 = \frac{1}{{g_{YM}}^2} < 0 \ ?$$

- **Based on *(p,q)* 5-brane web**

$$\frac{\partial F}{\partial \phi_i} = (\text{Monopole tension})$$
$$= (\text{Area of D3-brane in the web})$$

- **Global symmetry at UV fixed point**

  Add "non-perturbative terms" to the IMS prepotential so that the prepotential is invariant under the Weyl group of the expected global symmetry at UV fixed point

Plan of This Talk

✓§1. Introduction

§2. Complete prepotential from (p,q) 5-brane web

§3. Complete prepotential from global symmetry

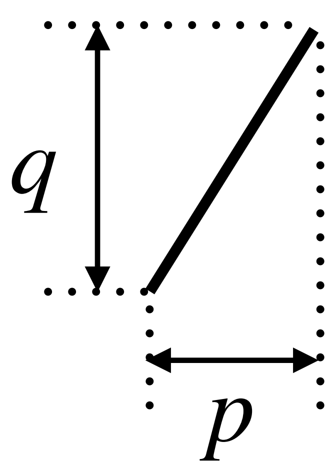
§4. Higher rank generalization

§5. Conclusion

(p, q) 5-brane web diagram

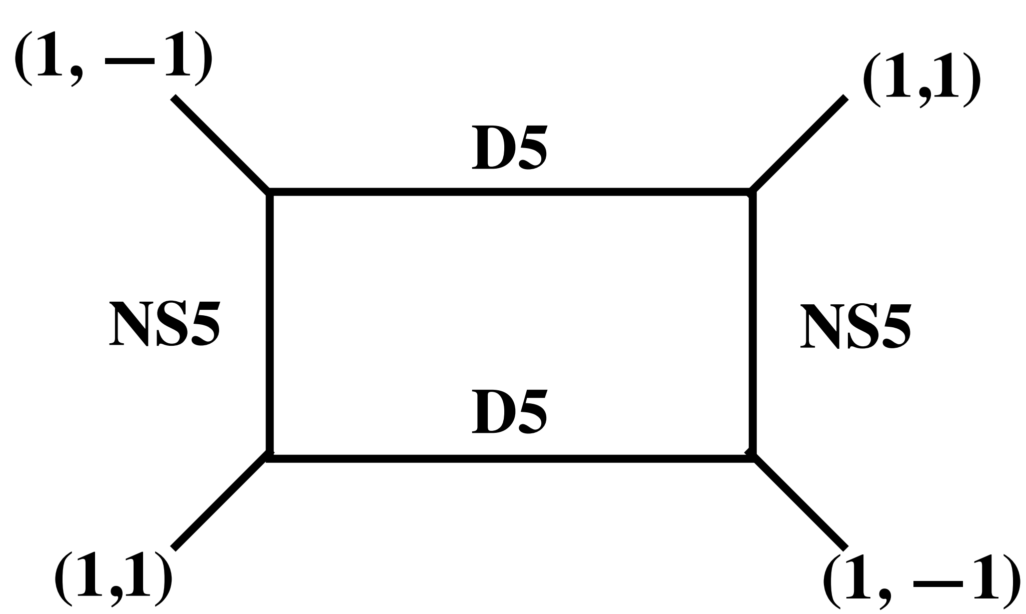
[Aharony, Hanany '97]
[Aharony, Hanany, Kol '97]

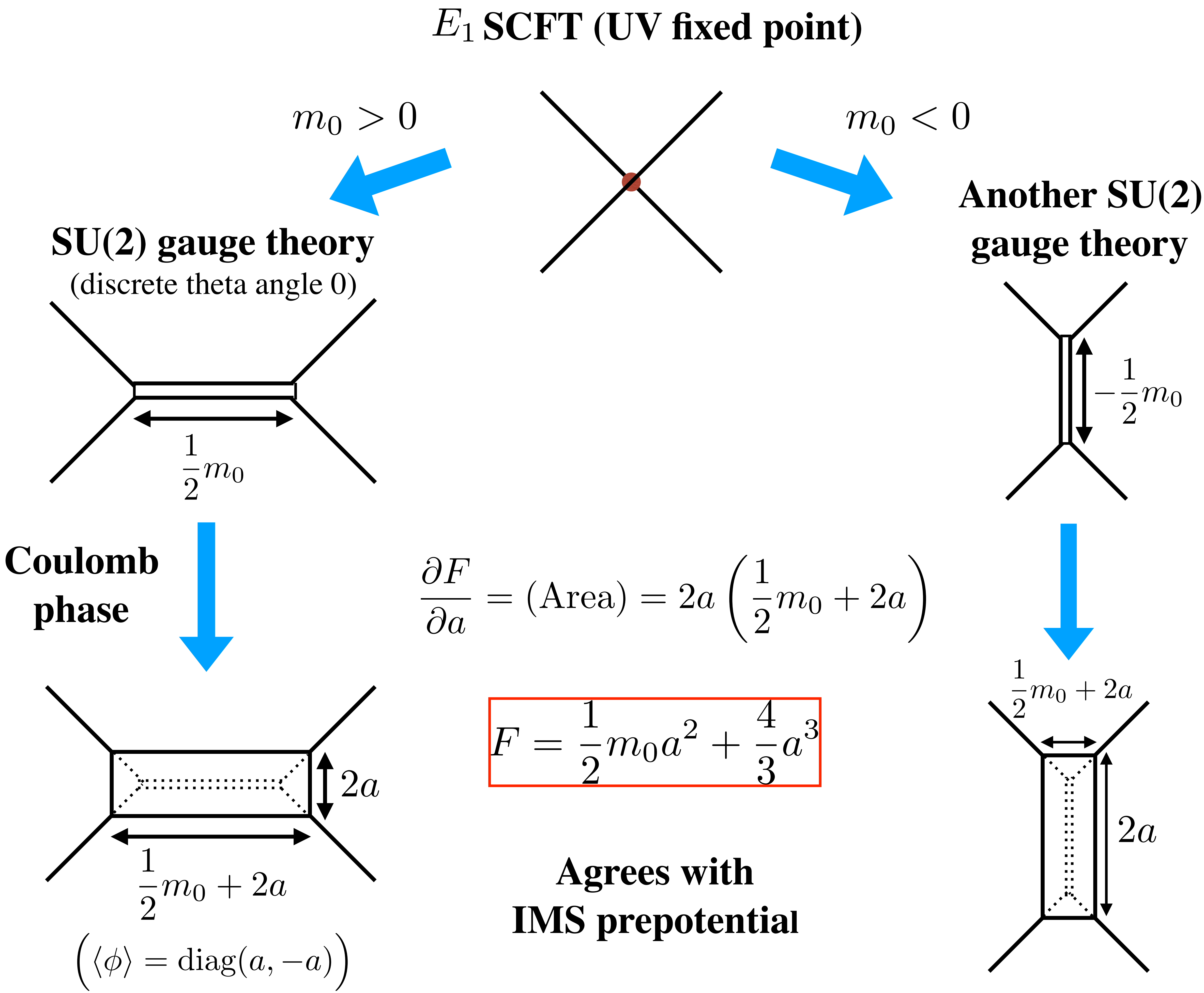
	0	1	2	3	4	5	6	7	8	9
5-brane	—	—	—	—	—	web		·	·	·



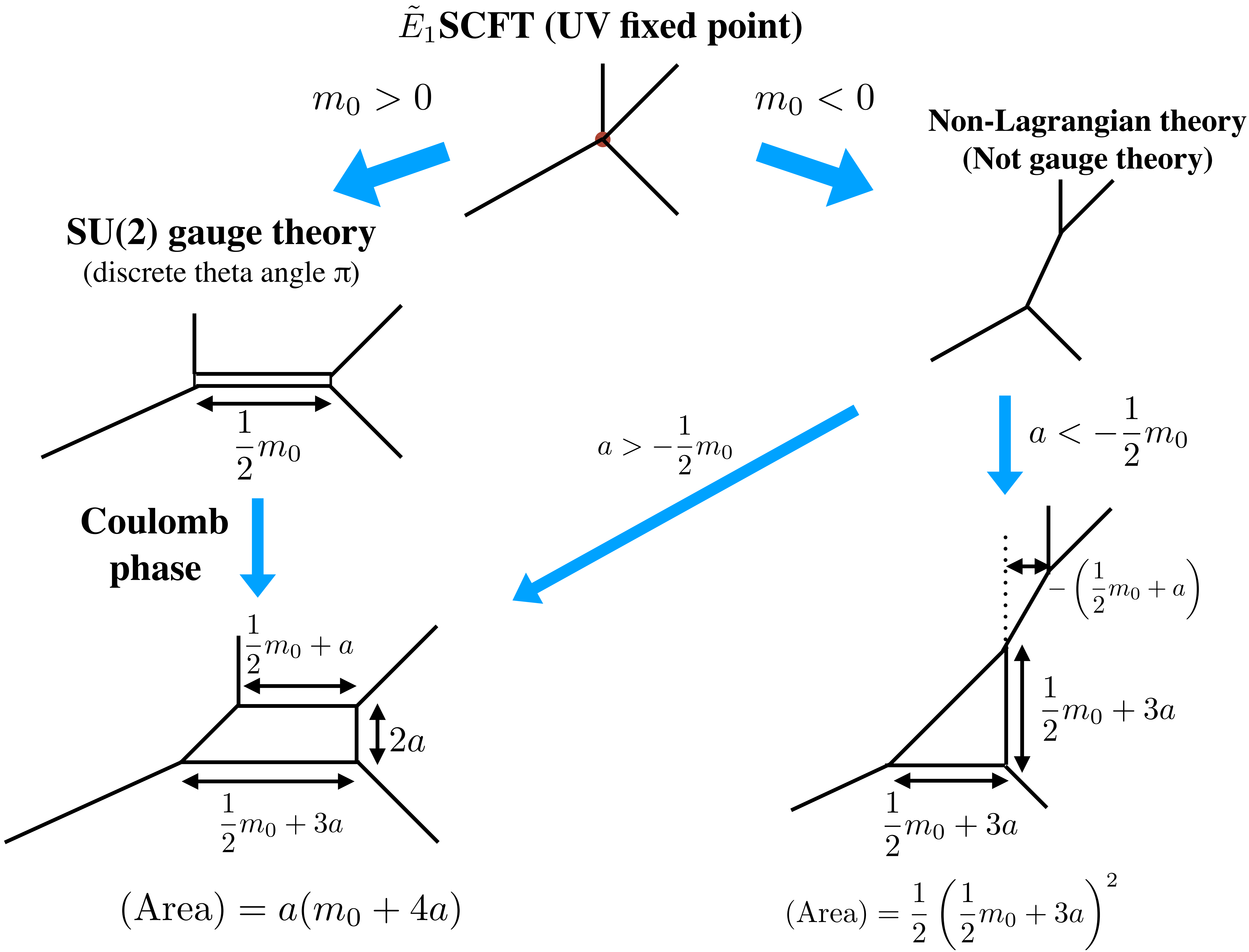
(p,q) 5-brane = p D5-brane + q NS5-brane

(1,0) 5-brane = D5 brane
(0,1) 5-brane = NS5 brane





E_1 SCFT (UV fixed point)
$m_0 > 0$
$m_0 < 0$
SU(2) gauge theory
(discrete theta angle 0)
Another SU(2) gauge theory
$\frac{1}{2}m_0$
$-\frac{1}{2}m_0$
Coulomb phase
$\frac{\partial F}{\partial a} = (\text{Area}) = 2a\left(\frac{1}{2}m_0 + 2a\right)$
$\boxed{F = \frac{1}{2}m_0 a^2 + \frac{4}{3}a^3}$
Agrees with IMS prepotential
$2a$
$\frac{1}{2}m_0 + 2a$
$\left(\langle\phi\rangle = \mathrm{diag}(a, -a)\right)$
$\frac{1}{2}m_0 + 2a$
$2a$


$\tilde{E}_1$ SCFT (UV fixed point)
$m_0 > 0$
$m_0 < 0$
Non-Lagrangian theory
(Not gauge theory)
SU(2) gauge theory
(discrete theta angle π)
$\frac{1}{2}m_0$
$a > -\frac{1}{2}m_0$
$a < -\frac{1}{2}m_0$
Coulomb phase
$\frac{1}{2}m_0 + a$
$2a$
$\frac{1}{2}m_0 + 3a$
$-\left(\frac{1}{2}m_0 + a\right)$
$\frac{1}{2}m_0 + 3a$
$\frac{1}{2}m_0 + 3a$
$(\text{Area}) = a(m_0 + 4a)$
$(\text{Area}) = \frac{1}{2}\left(\frac{1}{2}m_0 + 3a\right)^2$

Complete prepotential for $\tilde{E}_1$ theory

c.f. [Morrison, Seiberg '97]

$$F = \underbrace{\frac{1}{2}m_0 a^2 + \frac{4}{3}a^3}_{\text{IMS prepotential}} + \underbrace{\frac{1}{6}\left[\!\left[\, a + \frac{1}{2}m_0 \,\right]\!\right]^3}_{\text{Non-perturbative correction}} + (m_0^3 \text{ terms}) \qquad a \geq 0$$

IMS prepotential

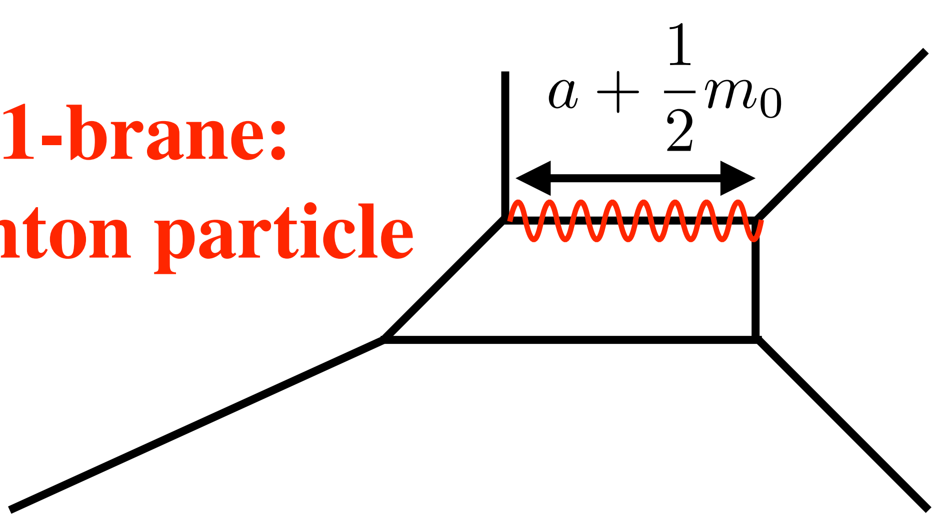
"Non-perturbative correction" from "instanton particle"

(BPS particle with instanton charge 1 at Coulomb phase)

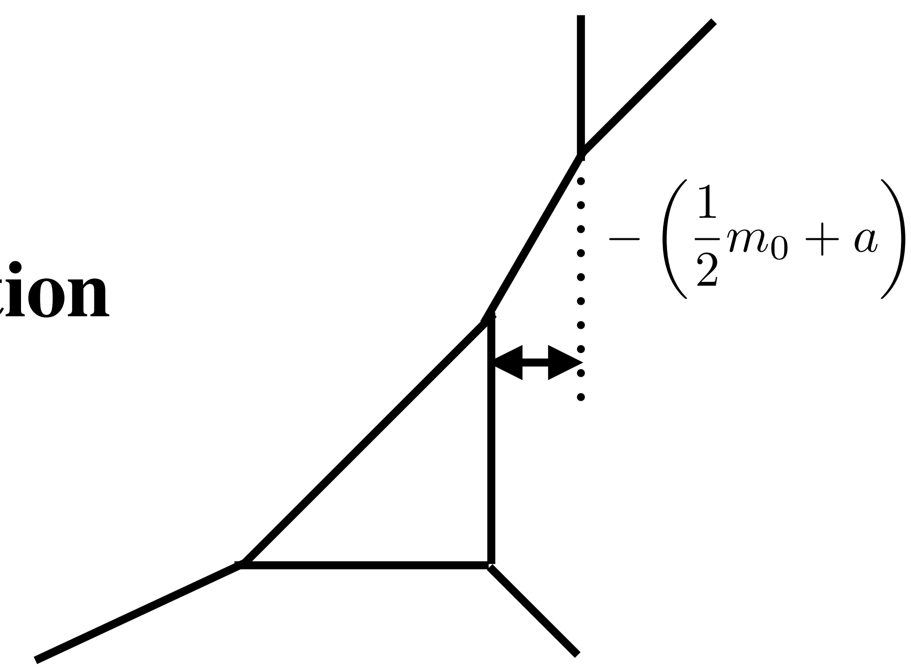
$$\left[\!\left[\, x \,\right]\!\right] := x\,\theta(-x) = \begin{cases} 0 & (x \geq 0) \\ x & (x < 0) \end{cases}$$

c.f. [Closset, del Zotto, Saxena '18]

D1-brane: instanton particle



Flop Transition



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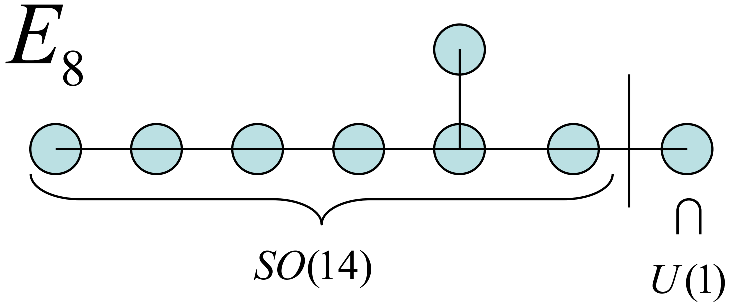
Global symmetry at UV fixed point for SU(2) gauge theory with N_f flavor

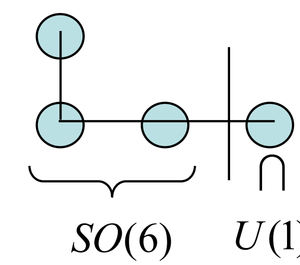
[Seiberg '96]

$$SO(2N_f) \times U(1)_I \subset E_{N_f+1}$$

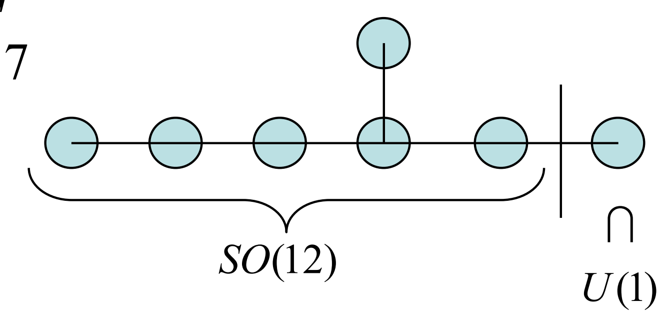
E_8



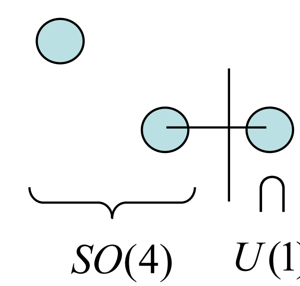
$E_4 = SU(5)$



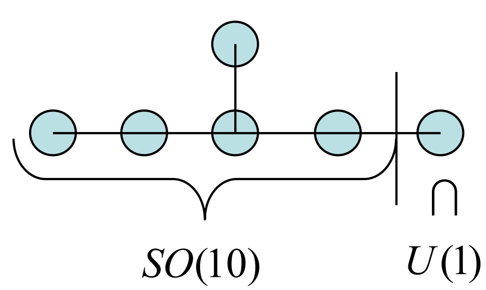
E_7



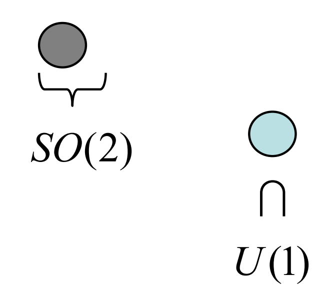
$E_3 = SU(2) \times SU(3)$



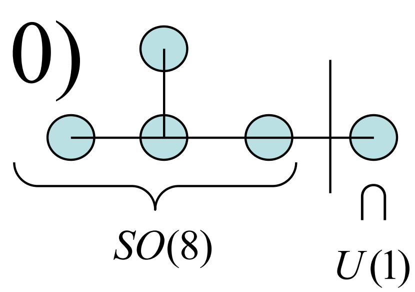
E_6



$E_2 = U(1) \times SU(2)$

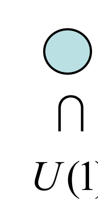


$E_5 = SO(10)$



$E_1 = SU(2)$

$\tilde{E}_1 = U(1)$



How can we see the global symmetry $E_1 = SU(2)$ from prepotential?

Weyl reflection of E_1

[Aharony, Hanany, Kol '97]

$$m_0 \to -m_0$$
$$a \to a + \frac{1}{4} m_0$$

"Invariant Coulomb branch parameter": $\tilde{a} = a + \frac{1}{8} m_0$

[Mitev, Pomoni, Taki, Yagi '14]

$$F(a) = \frac{1}{2} m_0 a^2 + \frac{4}{3} a^3$$
$$= \frac{4}{3} \tilde{a}^3 - \frac{1}{2} {m_0}^2 \tilde{a} + \cancel{\frac{1}{96} {m_0}^3}$$

Prepotential is invariance under the Weyl reflection of E_1

SU(2) with $N_f = 1$ flavor, $E_2 = SU(2) \times U(1)$

Weyl Reflection of E_2: $(x, y, \tilde{a}) \to (-x, y, \tilde{a})$

$$x := \frac{1}{4}m_0 + \frac{1}{4}m_1, \quad y := -\frac{1}{4}m_0 + \frac{7}{4}m, \quad \tilde{a} = a + \frac{1}{7}m_0$$

$$F = \frac{7}{6}\tilde{a}^3 - \left(x^2 + \frac{1}{7}y^2\right)\tilde{a}$$

$$+ \frac{1}{6}\llbracket \tilde{a} + \frac{4}{7}y \rrbracket^3 + \frac{1}{6}\llbracket \tilde{a} + x - \frac{3}{7}y \rrbracket^3$$

IMS prepotential

$\llbracket a \pm m \rrbracket^3$

$$+\frac{1}{6}\llbracket \tilde{a} - x - \frac{3}{7}y \rrbracket^3$$

Non-perturbative correction

$\llbracket a + \frac{1}{2}(m_0 - m) \rrbracket^3$

Agree with the complete prepotential computed from the area

[H.Hayashi, S.S.Kim, K.Lee, F.Yagi '17]
c.f. [Morrison, Seiberg '97]

Complete Prepotential for 5d rank 1 SCFT

(SU(2) with N_f flavor)

$$F = \frac{8-N_f}{6}\tilde{a}^3 + \left(\frac{1}{8-N_f}{m_0}^2 + \sum_{k=1}^{N_f}{m_k}^2\right)\tilde{a} + \frac{1}{6}\sum_{\mathbf{w}\in\text{weight of } E_{N_f+1}} \llbracket\, \tilde{a} + \mathbf{w}\cdot\mathbf{m} \,\rrbracket^3$$

$$\tilde{a} := a + \frac{1}{8-N_f}m_0$$

e.g. N_f = 7

$$F = \frac{1}{6}\tilde{a}^3 - \frac{1}{2}\sum_{k=0}^{7}{m_k}^2\tilde{a} + \frac{1}{6}\sum_{\substack{\{s_i=\pm1\}\\ \sum_i x_i = 0 \bmod 4}} \llbracket\, \tilde{a} + \frac{1}{2}\sum_{k=0}^{7} s_k m_k \,\rrbracket^3$$

$$+ \frac{1}{6}\sum_{\substack{s_1=\pm1\\ x_2=\pm1}}\ \sum_{0\le i<j\le 7} \llbracket\, \tilde{a} + s_1 m_i + s_2 m_j \,\rrbracket^3 + \frac{1}{6}\llbracket\, \tilde{a} \,\rrbracket^3 \times 8$$

248 terms

Observation 1

$$F^{(\mathrm{Complete})} \; = \; F^{(\mathrm{IMS})} \quad \text{for } m_0 \gg |a|, |m_f|$$

IMS prepotential is reproduced in the weak coupling region

Observation 2

$$F^{(\mathrm{Complete})} \; = \; F^{(\mathrm{IMS})} \quad \text{for } a \gg |m_0|, |m_f|$$

(The case $m_0 = m_f = 0$ is included)

IMS prepotential is reliable up to CFT point

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Complete Prepotential for 5d Sp(2) with N_f= 9 flavor

$$
\begin{aligned}
\mathcal{F} =& \frac{1}{6}(\tilde{a}_1^3 - 2\tilde{a}_2^3) + \tilde{a}_1\tilde{a}_2^2 - \frac{1}{2}\sum_{f=0}^{9} m_f^2(\tilde{a}_1+\tilde{a}_2) \\
&+ \frac{1}{6}\sum_{0\le i<j\le 9}\sum_{\substack{s_1=\pm 1\\ s_2=\pm 1}} \Big\| \tilde{a}_1 + s_1 m_i + s_2 m_j \Big\|^3 + \frac{1}{6}\sum_{i=0}^{9}\sum_{s=\pm 1}\Big\| \tilde{a}_2 + s\, m_i \Big\|^3 \\
&+ \frac{1}{6}\sum_{\substack{\{s_i=\pm 1\}\\ \sum_i s_i = 0 \bmod 4}} \Big\| \tilde{a}_1 + \tilde{a}_2 + \frac{1}{2}\sum_{i=0}^{9} s_i m_i \Big\|^3 \\
&+ \frac{1}{6}\sum_{0\le i_1<i_2<i_3<i_4<i_5\le 9}\;\sum_{\{s_k=\pm 1\}} \Big\| 2\tilde{a}_1 + \tilde{a}_2 + \sum_{k=1}^{5} s_k m_{i_k} \Big\|^3 .
\end{aligned}
$$

$$\tilde{a}_1 = a_1 + m_0, \qquad \tilde{a}_2 = a_2$$

Observation 1'

$$F^{(\mathrm{Complete})} = F^{(\mathrm{IMS})} \quad \text{for } m_0 \gg |a_i|, |m_f|$$

IMS prepotential is reproduced in the weak coupling region

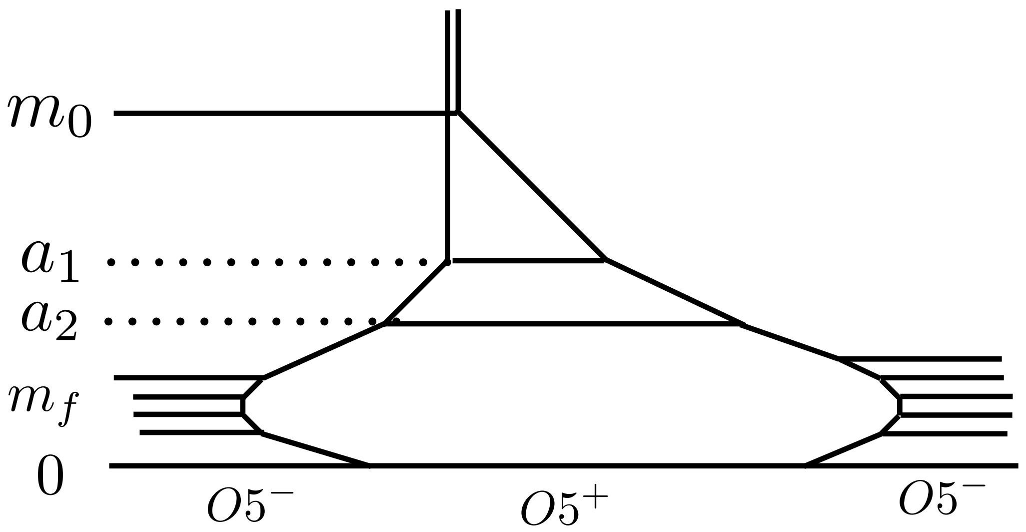
Observation 2'

$$F^{(\mathrm{Complete})} = F^{(\mathrm{IMS})} - \frac{1}{6}(a_2 - m_0)^3 \qquad \text{for } a_1 > a_2 \gg |m_0|, |m_f|$$

$$\left(\text{Especially, } F^{(\mathrm{Complete})} = F^{(\mathrm{IMS})} - \frac{1}{6}{a_2}^3 \qquad \text{for } m_0 = m_f = 0 \right)$$

IMS prepotential is NOT reliable up to CFT point!

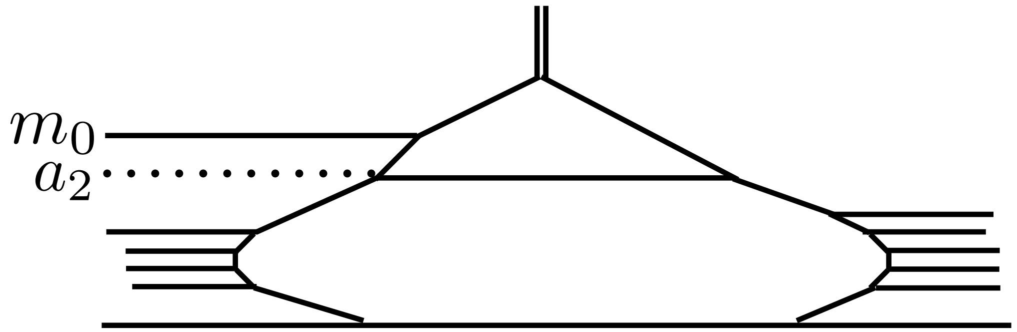
Diagramatic interpretation



$$m_0 > a_1 > a_2 (> m_f)$$

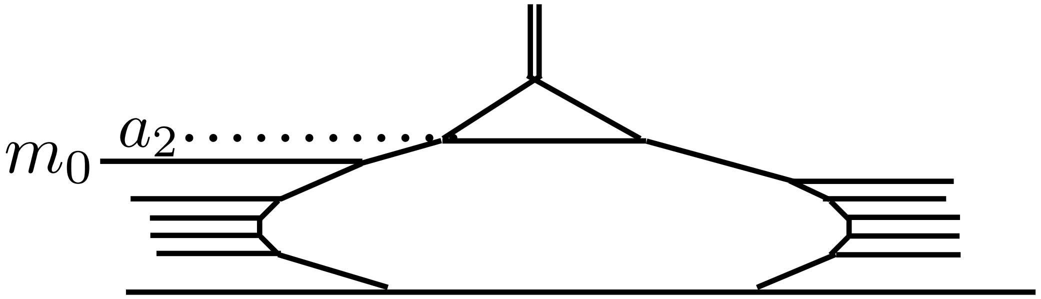
IMS prepotential is valid

No modification for prepotential



$$a_1 > m_0 > a_2 (> m_f)$$

Need to add non-perturbative correction $-\frac{1}{6}(a_2 - m_0)^3$



$$a_1 > a_2 > m_0 (> m_f)$$

"CFT phase": IMS prepotential is not valid

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Conclusion

We should add non-perturbative correction to the IMS prepotential in order to understand the CFT realized at the UV fixed point

Thank you!